ENG.303: Numerical Methods for Aerospace Engineers
http://mapping.mit.edu/my-class-webpage/
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1 Course Objectives

Students successfully completing ENG.303 should have:

1. A conceptual understanding of computational methods commonly used for analysis and design of aerospace systems.
2. A working knowledge of computational methods including experience implementing them for model problems drawn from aerospace engineering applications.
3. A basic foundation in theoretical techniques to analyze the behavior of computational methods.

2 Measurable Outcomes

The subject is divided into four sections:

- Integration of Systems of Ordinary Differential Equations (ODEs)
- Finite Difference and Finite Volume Methods for Partial Differential Equations (PDEs)
- Finite Element Methods for Partial Differential Equations
- Probabilistic Simulation

For each of these sections, the measurable outcomes are described below. Specifically, a student successfully completing ENG.303 will be able to:

Integration Methods for ODEs

1. (a) Describe the Adams-Bashforth, Adams-Moulton, and backwards differentiation families of multi-step methods, (b) Describe the form of the Runge-Kutta family of multi-stage methods, and (c) Explain the relative computational costs of multi-step versus multi-stage methods.
2. (a) Explain the concept of stiffness of a system of equations, and (b) describe how it impacts the choice of numerical method for solving the equations.
3. (a) Explain the differences and relative advantages between explicit and implicit methods to integrate systems of ordinary differential equations, and (b) for nonlinear systems of equations, explain how a Newton-Raphson can be used in the solution of an implicit method.

4. (a) Define a convergent method, (b) Define a consistent method, (c) Explain what (zero) stability is, and (d) Demonstrate an understanding of the Dahlquist Equivalence Theorem by describing the relationship between a convergent method, consistency, and stability.
5. Determine if a multi-step method is stable and consistent.
6. (a) Define global and local order of accuracy for a ODE integration method, (b) describe the relationship between global and local order of accuracy, and (c) calculate the local order of accuracy for a given method using a Taylor series analysis.
7. (a) Define eigenvalue stability, and (b) determine the stability boundary for a multi-step or multi-stage method applied to a linear system of ODEs.
8. Recommend an appropriate ODE integration method based on the features of the problem being solved.
9. Implement multi-step and multi-stage methods to solve a representative system of ODEs from an engineering application.

Finite Difference and Finite Volume Methods for PDEs

1. (a) Define the physical domain of dependence for a problem, (b) Define and determine the numerical domain of dependence for a discretization, and (c) Explain the CFL condition and determine the timestep constraints resulting from the CFL condition.
2. Determine the local truncation error for a finite difference approximation of a PDE using a Taylor series analysis.
3. Explain the difference between a centered and a one-sided (e.g. upwind) discretization.
4. Describe the Godunov finite volume discretization of two-dimensional convection on an unstructured mesh.
5. Perform an eigenvalue stability analysis of a finite difference approximation of a PDE using either Von Neumann analysis or a semi-discrete (method of lines) analysis.
6. Implement a finite difference or finite volume discretization to solve a representative PDE (or set of PDEs) from an engineering application.

Finite Element Methods for PDEs

1. (a) Describe how the Method of Weighted Residuals (MWR) can be used to calculate an approximate solution to a PDE, (b) Describe the differences between MWR, the collocation method, and the least-squares method for approximating a PDE, and (c) Describe what a Galerkin MWR is.
2. (a) Describe the choice of approximate solutions (i.e. the test functions or interpolants) used in the finite element method, and (b) Give examples of a basis for the approximate solutions in particular including a nodal basis for at least linear and quadratic solutions.
3. (a) Describe how integrals are performed using a reference element, (b) Explain how Gaussian quadrature rules are derived, and (c) Describe how Gaussian quadrature is used to approximate an integral in the reference element.

4. Explain how Dirichlet and Neumann boundary conditions are implemented for Laplace's equation discretized by the FEM.

5. (a) Describe how the FEM discretization results in a system of discrete equations and, for linear problems, gives rise to the stiffness matrix, and (b) Describe the meaning of the entries (rows and columns) of the stiffness matrix and of the right-hand side vector for linear problems.

Probabilistic Simulation

Based on pre- and co-requisite coursework, all students are expected to understand basic concepts of probability and random variables; probability densities and distribution functions; expected values and moments; quantiles; conditional probability; Bayes' rule. Students should also be familiar with the properties of various canonical distributions (e.g., uniform and Gaussian).

1. (a) Describe the process of Monte Carlo sampling from uniform distributions, (b) Describe how to generalize Monte Carlo sampling from uniform distributions to arbitrary univariate distributions, (c) use Monte Carlo simulation to propagate uncertainty through an ODE or PDE model.

2. (a) Describe what an estimator is, (b) Define the bias and variance of an estimator, (c) State unbiased estimators for mean and variance of a random variable, and for the probability of particular events.

3. (a) Define the standard error and sampling distribution of an estimator; (b) Give standard errors for sample estimators of mean, variance, and event probability; (c) Obtain confidence intervals for sample estimates of the mean, variance, and event probability; and (d) Numerically demonstrate Monte Carlo convergence and assess agreement with the error estimates.

4. (a) Describe the importance sampling approach to Monte Carlo variance reduction, and identify attributes of a good biasing distribution for importance sampling; (b) Describe the control variate approach to Monte Carlo variance reduction and identify the attributes of an effective control variate; (c) Numerically demonstrate the impact of these variance reduction approaches.

5. (a) Describe stratified sampling for single input and multiple inputs, (b) Describe Latin Hypercube Sampling (LHS), and (c) Describe the benefits of LHS for nearly linear outputs in terms of the standard error convergence of the mean with the number of samples.

6. Describe methods for performing global sensitivity analyses.

3 Homework Problems

Homework problem sets will be given weekly to bi-weekly, with gaps for exams and project due dates. Problem sets will be due at the beginning of class on Wednesdays. Expected dates for the homeworks are as follows:

	Handed out	Due
HW1	Feb 15	Feb 22
HW2	Feb 22	Mar 1
HW3	Mar 22	Apr 5
HW4	Apr 5	Apr 12
HW5	Apr 26	May 3

Expected Homework Dates

Assignments will consist of a combination of look-ahead and look-back problems. Look-ahead problems will be based on lecture notes and textbook references that will be provided ahead of time. At the end of the semester, the numerical scores received in the homeworks will be averaged to determine an overall homework letter grade.

Late homeworks will *not* be accepted unless there are extenuating circumstances.

4 Projects

You will complete three programming projects this semester. The projects will focus on applying numerical algorithms to aerospace applications. It is recommended that programming be done in MATLAB. Expected dates for the projects are as follows:

	Handed out	Due
Project 1	March 1	March 15
Project 2	April 12	April 26
Project 3	May 3	May 10

Expected Project Dates

The project assignments will distributed at least one week prior to the due dates. No homeworks will be given during the week the projects are due. Each project will be assigned a letter grade based on the standard MIT letter grade descriptions (see Section 9).

5 Recitations

An optional recitation session will be scheduled if desired (to be discussed at first lecture).

6 Office Hours

Office hours will be scheduled shortly. The locations and times will be announced in class and posted on the course website.

7 Homework and Project Collaboration

While discussion of the homework and projects is encouraged among students, the work submitted for grading must represent your understanding of the subject matter. Significant help from other students or other sources should be noted. You are expected to write your own codes and your own reports for the projects.

8 Exams

There will be an oral mid-term exam and a written final exam. The mid-term exam will be during the period March 20–22 (there will be no class Wednesday March 22). Specific times for the midterm will be scheduled 2–3 weeks prior based on preferences from each student. The final exam will be held during Final Exam Week. Each exam will be assigned a letter grade based on the standard MIT letter grade descriptions (see Section 9).

9 Course Grade

The subject total grade will be based on the letter grades from the homework, projects, and exams. The weighting of the individual letter grades is as follows:

- Homework letter grade: Homeworks (combined) compose 1/8 of the total grade.
- Project letter grades: Each project is 1/8 of the total grade.
- Midterm exam letter grade: 1/4 of the total grade.
- Final exam letter grade: 1/4 of the total grade.

For the subject letter grade, we adhere to the MIT grading guidelines which give the following description of the letter grades:

A: Exceptionally good performance demonstrating a superior understanding of the subject matter, a foundation of extensive knowledge, and a skillful use of concepts and/or materials.

B: Good performance demonstrating capacity to use the appropriate concepts, a good understanding of the subject matter, and an ability to handle the problems and materials encountered in the subject.

C: Adequate performance demonstrating an adequate understanding of the subject matter, an ability to handle relatively simple problems, and adequate preparation for moving on to more advanced work in the field.

D: Minimally acceptable performance demonstrating at least partial familiarity with the subject matter and some capacity to deal with relatively simple problems, but also demonstrating deficiencies serious enough to make it inadvisable to proceed further in the field without additional work.

10 Textbooks

Notes will be distributed. Reference texts will be recommended for specific topics as needed. Some useful texts include:

Lloyd N. Trefethen, Finite Difference and Spectral Methods for Ordinary and Partial Differential Equations, unpublished text, 1996, available at
`http://www.comlab.ox.ac.uk/nick.trefethen/pdetext.html`

Arieh Iserles, A First Course in the Numerical Analysis of Differential Equations, 2nd edition, Cambridge UP, 2008.

K. W. Morton & D. F. Mayers, Numerical Solution of Partial Differential Equations, Cambridge UP, 1995.

J. N. Reddy, An Introduction to the Finite Element Method, 3rd edition, McGraw-Hill, New York, 2005.

Michael T. Heath, Scientific Computing: An Introductory Survey, 2nd edition, McGraw-Hill, New York, 2002.

Morris H. DeGroot & Mark J. Schervish, Probability and Statistics, 3rd edition, Addison-Wesley, 2001.